Lateral load distributions on grouped piles from dynamic pile-to-pile interaction factors

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ABSTRACT

The load distributions of the grouped piles under lateral loads acting from one side of the pile cap could be approximately modeled using the elasticity equations with the assumptions that the underground structure is rigid enough to sustain the loads, and only small deformations of the soils are yielded. Variations of the soil-pile interactions along the depths are therefore negligible for simplicity. This paper presents the analytical modeling using the dynamic pile-to-pile interaction factors for $2 \times 2$ and $2 \times 3$ grouped piles. The results were found comparative to the experimental and numerical results of other studies. Similar to others’ finding, it was shown that the leading pile could carry more static loads than the trailing pile does. For the piles in the perpendicular direction with the static load, the loads would distribute symmetrically with the centerline whereas the middle pile always sustains the smallest load. For steady-state loads with operating frequencies up to 30 Hz, the pile load distributions would vary significantly with the frequencies. It is interesting to know that designing the pile foundation needs to be cautioned for steady state vibrations as the problem of machine foundation. However for transient loads or any harmonic loads acting upon relatively higher frequencies, the pile loads could be regarded as uniformly distributed. It is hoped that the numerical results of this paper could be helpful to the design practice of pile foundation.

Key words: lateral load distributions, grouped piles, pile-to-pile interactions
INTRODUCTION

Many researchers have investigated the mechanism of lateral load distribution on the piles in the past decades. It is known that the shadowing effect of the grouped piles (see Figure 1) is rather important for the design of the laterally loaded pile foundation (Reese et al. 1974; Holloway et al. 1981; Brown et al. 1987, 1988, 2001; Ruesta and Townsend 1997; McVay et al. 1995, 1998; Rollins et al. 1998, 2005; Zhang et al. 1999; Ng et al. 2001; Chae et al. 2004 and Ilyas et al. 2004). The shadowing effect making the stress zone out of a pile and the surrounding soils can produce a passive wedge along the pile. This wedge will interact with others and then reduce the soil’s resistances. In consequence, the soils along the trailing piles would be more significantly affected than those along the leading piles; individual pile loads are thus different. To investigate this effect, scaled model test and centrifuge test are often used to monitor these phenomena. Studying the 3\(\frac{D}{2}\) scaled model piles where the pile-to-pile spacing was kept three times as wide as the pile diameter, Brown et al. (1988) had pointed out that the distributions of the ultimate loads could be 45% for leading piles, 32% for middle piles and 23% for trailing piles. Considering the grouped pile effects, a multiplying factor was then suggested to reduce the loads in the \(p-y\) curve analysis. On the other hand, McVay et al. (1995) have conducted the centrifuge tests on 2\(\frac{D}{2}\) piles where the pile-to-pile spacing was kept twice as wide as the pile diameter. It was reported that the total load ratio of the leading piles is 60%, and that of the trailing piles is 40%. Their subsequent studies (1998) further showed that the pile orientations and the pile-to-pile spacing would be the main factors in affecting these distributions. Zhang et al. (1999) had confirmed their observations using the 3D FEM analysis. Recently, Ng et al. (2001) have demonstrated that the soils and the pile cap connection as well as the loading itself could also affect the lateral load distributions for the pile foundations. In general, the variations of the pile loads on a grouped pile foundation strongly depend on the interactions between the soils and the
structure.

Presuming that the relative deformations between the piles and the soils are small, the system can thus preserve elasticity. The pile-to-pile interaction factor (Poulos, 1968, 1971) was defined as the pile displacement ratio for the arbitrary two piles due to the same load acting on one of the piles. In this study, this factor is often referred in calculating the grouped pile stiffness affected by the pile-to-pile interaction effects. Using the classical treatise by Morse and Ingard (1968) on asymptotic cylindrical wave model and the interaction model first proposed by Kaynia and Kausel (1982) for two horizontally loaded piles, Dobry and Gazetas (1988) have suggested the simplified vertical and horizontal dynamic pile-to-pile interaction factors for piles subjected to steady-state loads. Their formulas could be approximated as follows,

\[ \alpha_v = \left( \frac{S}{r_0} \right)^{1/2} \exp \left( -\frac{\beta \omega S}{V_s} \right) \exp \left( -\frac{i \omega S}{V_s} \right) \]

\[ \alpha_h(\theta, S) \equiv \alpha_h(0^\circ, S) \cos^2 \theta + \alpha_h(90^\circ, S) \sin^2 \theta \]

\[ \alpha_h(90^\circ, S) \approx 0.75 \times \alpha_v(S) \]

\[ \alpha_h(0^\circ, S) \approx \left( \frac{S}{r_0} \right)^{1/2} \exp \left( -\frac{\beta \omega S}{V_{La}} \right) \exp \left( -\frac{i \omega S}{V_{La}} \right) \]

where \( \alpha_v \) and \( \alpha_h \) are respectively the vertical and the horizontal pile-to-pile interaction factors, \( S \) is the pile-to-pile spacing, \( r_0 \) is the pile radius, \( \omega \) is the operation frequency (rad/sec), \( \beta \) is material damping of the soil, and \( V_s \) is shear wave velocity of the soil, \( \theta \) is the angle between the line of loading and the line through two piles, \( V_{La} \) is the so-called ‘Lysmer’s analogue’ wave velocity for soil \( (= 3.4 \frac{V_s}{\pi(1-\nu)} \) where \( \nu \) is the Possion’s ratio of soil). It is a fictitious wave velocity to approximate the apparent phase velocity for the compression-extension waves in the same direction of the force. It is important to know that Eq. (1) was modeled based on in-plane SV-wave motions due the vertical force.
excitations. Eq. (2) was modeled based on two types of waves with the assumed apparent velocities emanating from an oscillating pile under horizontal loads. Assuming that the wave traveling perpendicularly to the horizontal force is governed by out-of-plane SH-wave, the corresponding interaction factor could be approximated from the vertical one. Accordingly Eq. (3) follows the suggestion of Gazetas (1991) with a multiplying factor of 0.75. Eq. (4) was obtained presuming that the waves traveling in the direction of horizontal force were mainly the compression-extension waves with an apparent phase velocity approximately equal to the Lysmer’s analog velocity. With these equations, the static interaction factors can be obtained by taking zero frequency, which yields:

\[ \alpha_v = (S/r_o)^{-1/2} \]

\[ \alpha_h(\theta, S) = (S/r_o)^{-1/2} (\cos^2 \theta + 0.75 \sin^2 \theta) \]

Note that the above equations have been lately revised by Makris and Gazetas (1992) and discussed in Mylonakis and Gazetas (1999). Essentially, the symbol \( S \) used in the exponential terms is replaced by \( S - r_0 \) in order to satisfy the interaction at a distance of the pile radius. For real applications, using the above equations instead of their revised ones would only result computing significance in the vicinity around the pile shaft. It is necessary to point out that the impedance of a pile group should be strongly affecting by the cap-pile-soil interactions. Ignoring the cap-pile-soil interactions and the soil’s inelasticity shall produce conservative results. As the piles must interact through the soils, the actual pile loads found at a distance from the source should be less than those predicted from this approximation. It should be known that this simplification has some shortcomings especially when relatively large pile foundations were encountered.

Static conditions of using these formulas with the modifications that incorporate the stiffness and length of the pile elements into pile-to-pile interactions have been examined by Randolph (2003) and are discussed in a recent text by Salgado (2006). Although the
stiffening of the piles in the soils is ignored in Eqs. (1)–(4) in this study, one must know that the simplified equations could provide very accurate results for stiff piles in homogeneous soil. The accuracy of the method was reported gradually deteriorates when dealing with strongly inhomogeneous soil or piles of small slenderness (Mylonakis, 1995).

For simplicity of modeling and based on the assumptions that the pile-cap connection is rigid and the cap-pile-soil interactions are negligible, the authors (2003) have used the above equations to monitor the pile load distributions of vertical and horizontal loads acting at the centre of the foundation. It was revealed the vertical load would distribute symmetrically at the foundation. For the horizontal loading, load distributions would vary for piles in the longitudinal and transverse directions with respect to the loads. Table 1 indicates respectively the horizontal load distributions for 2 \( \square \) and 3 \( \square \) grouped piles with \( S/D = 3 \) (where \( D \) is the pile diameter). Further, Figure 2 illustrates the load distribution ratio spectra and the corresponding time histories for piles at a 3 \( \square \) pile group with \( S/D = 3 \). \( F_a, F_b, F_c \) and \( F_d \) represent respectively for the load ratios at the center pile, the corner piles, the edge piles perpendicular to the load direction, and the edge piles along the load direction. Note that the time-dependent functions are obtained by conducting the Fourier Transforms on load distribution ratio spectra. The load distribution ratios versus time could be used to depict approximately the load distribution nature for the foundation applied with arbitrary transient loadings. In Figure 2, it is clearly shown that the edge piles would sustain most of the static load. Variations of the load distributions were only conceived as significant at frequencies less than 30 Hz. It is necessary to point out that 10 Hz is usually the upper limit for realistic predictions in such problem. For steady state loads at relatively higher frequencies and the possible transient loading, the loads would uniformly distribute at the piles. The simple elasticity modeling can be reasonably extended for the complex problems where the lateral loads are acting from one side of the foundation. Also, the variations of the flexibilities
between the soils and the piles along the depths are ignored.

**MODELLING AND DERIVATIONS**

To approximate the load distributions for the lateral loads applied at one side of the pile foundation, an equal-size ‘mirror part’ of the actual pile foundation (see Figure 3 indicated with dash lines) is placed next to the actual foundation (solid lines) to yield an enlarged pile foundation. Assuming that the enlarged foundation has a horizontal load $P$ applied on its centre, the dynamic pile to-pile interactions of these piles can be easily computed using Eqs (1)–(4). According to the prescribed load symmetry, the counter-folded load distributions for the piles at the ‘mirror part’ of the foundation would be the same as those at the actual one. This implies that the actual part and the ‘mirror part’ of the enlarged foundation would respectively carry half of the total load. By simply assuming that total load distributions of the actual part of the enlarged foundation are similar to those of a single foundation applied with $1/2$ $P$ load laterally, the load distribution ratios computed for the actual part of the enlarged foundation could be doubled to approach the solutions for the later. The equations required to compute the load distributions for the $2 \Omega_2$ and $2 \Omega_3$ grouped piles are presented next. One may use the same procedures to compute the load distributions for arbitrary pile foundation under the lateral loads. As mentioned previously, double the size of the foundation does not necessarily double the foundation’s impedance. While more complexities should involve in solving the load distributions, application of this simple approximation requires further cautioned.

Assuming that the $2 \Omega_2$ pile foundations have the orientations as those shown in Figure 3, where $F_1$ and $F_2$ are respectively the loads at the trailing piles and the leading piles, $u_1$ and $u_2$ are the corresponding displacements of the piles, $N$ is the number of piles, $K_G$ is the dynamic impedance of the pile foundation, $K_{si}$ is the dynamic impedance of a single pile, and
$K'_{si}$ is the dynamic impedance of a single pile affected by pile-to-pile interactions, the corresponding formulas are listed as follows,

\[ K_{si} = \sum_{i=1}^{N} K'_{si} \]

for the trailing piles:

\[ K'_{si} = \frac{F_{si}}{u_{i}} \]

\[ u_{i} = \frac{F_{si}}{K_{s}} \times \left[ 1 + \alpha_{s}(90^\circ,S) + \alpha_{s}(0^\circ,S) + \alpha_{s}(45^\circ,\sqrt{2}S) \right] 
+ \frac{F_{si}}{K_{s}} \times \left[ \alpha_{s}(0^\circ,S) + \alpha_{s}(45^\circ,\sqrt{2}S) + \alpha_{s}(0^\circ,2S) + \alpha_{s}(\tan^{-1}1/2,\sqrt{5}S) \right] \]

for the leading piles:

\[ K_{s} = \frac{F_{s}}{u_{2}} \]

\[ u_{2} = \frac{F_{s}}{K_{s}} \times \left[ \alpha_{s}(0^\circ,S) + \alpha_{s}(45^\circ,\sqrt{2}S) + \alpha_{s}(0^\circ,2S) + \alpha_{s}(\tan^{-1}1/2,\sqrt{5}S) \right] 
+ \frac{F_{s}}{K_{s}} \times \left[ 1 + \alpha_{s}(90^\circ,S) + \alpha_{s}(0^\circ,3S) + \alpha_{s}(\tan^{-1}1/3,\sqrt{10}S) \right] \]

Since the pile cap deforms rigidly, one can infer that all the pile displacements are about equal, where $u_{1} = u_{2}$. By equating Eqs. (9) and (11), the following equations could be obtained.

\[ A_{1} \cdot F_{1} + A_{2} \cdot F_{2} = 0 \]

where

\[ A_{1} = 1 + \alpha_{s}(90^\circ,S) - \alpha_{s}(0^\circ,2S) - \alpha_{s}(\tan^{-1}1/2,\sqrt{5}S) \]

\[ A_{2} = \alpha_{s}(0^\circ,S) + \alpha_{s}(45^\circ,\sqrt{2}S) + \alpha_{s}(0^\circ,2S) + \alpha_{s}(\tan^{-1}1/2,\sqrt{5}S) 
- 1 - \alpha_{s}(90^\circ,S) - \alpha_{s}(0^\circ,3S) - \alpha_{s}(\tan^{-1}1/3,\sqrt{10}S) \]

For total load of 100%, $F_{1}$ and $F_{2}$ can be treated as the load distribution ratios and solved from Eqs. (12) and (13).

\[ 4 \cdot F_{1} + 4 \cdot F_{2} = 1 \]

Similarly, the 2 piled foundations have the orientations as those shown in Figure 3, where $F_{1}$, $F_{2}$ and $F_{3}$ are the loads respectively at the trailing piles, the middle piles and the
leading piles. Corresponding equations are as follows,

\[ K_{a1} = \frac{F_1}{u_1}; \quad K_{a2} = \frac{F_2}{u_2}; \quad K_{a3} = \frac{F_3}{u_3} \]

for the trailing piles:

\[
\begin{align*}
  u_1 &= \frac{F_1}{K_{a1}} \times [1 + \alpha_s(90^\circ, S) + \alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S)] \\
  &\quad + \frac{F_2}{K_{a2}} \times [\alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) + \alpha_s(20^\circ, 2S) + \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S)] \\
  &\quad + \frac{F_3}{K_{a3}} \times [\alpha_s(0^\circ, 2S) + \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S) + \alpha_s(0^\circ, 3S) + \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S)]
\end{align*}
\]

for the middle piles:

\[
\begin{align*}
  u_2 &= \frac{F_1}{K_{a1}} \times [\alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) + \alpha_s(0^\circ, 2S) + \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S)] \\
  &\quad + \frac{F_2}{K_{a2}} \times [1 + \alpha_s(90^\circ, S) + \alpha_s(0^\circ, 3S) + \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S)] \\
  &\quad + \frac{F_3}{K_{a3}} \times [\alpha_s(0^\circ, 4S) + \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{17}S)]
\end{align*}
\]

for the leading piles:

\[
\begin{align*}
  u_3 &= \frac{F_1}{K_{a1}} \times [\alpha_s(0^\circ, 2S) + \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S) + \alpha_s(0^\circ, 3S) + \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S)] \\
  &\quad + \frac{F_2}{K_{a2}} \times [\alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) + \alpha_s(0^\circ, 4S) + \alpha_s(\tan^{-1} \frac{1}{4} \sqrt{17}S)] \\
  &\quad + \frac{F_3}{K_{a3}} \times [1 + \alpha_s(90^\circ, S) + \alpha_s(0^\circ, 5S) + \alpha_s(\tan^{-1} \frac{1}{5} \sqrt{26}S)]
\end{align*}
\]

Similar, since that \( u_1 = u_2; \quad u_2 = u_3 \), the following equations are obtainable.

\[ A_1 \cdot F_1 + A_2 \cdot F_2 + A_3 \cdot F_3 = 0 \]

where

\[ A_1 = 1 + \alpha_s(90^\circ, S) - \alpha_s(0^\circ, 2S) - \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S) \]

\[ A_2 = \alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) + \alpha_s(0^\circ, 2S) + \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S) \]

\[ -1 - \alpha_s(90^\circ, S) - \alpha_s(0^\circ, 3S) - \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S) \]

\[ A_3 = \alpha_s(0^\circ, 2S) + \alpha_s(\tan^{-1} \frac{1}{2} \sqrt{5}S) + \alpha_s(0^\circ, 3S) + \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S) \]

\[ - \alpha_s(0^\circ, S) - \alpha_s(45^\circ, \sqrt{2}S) - \alpha_s(0^\circ, 4S) - \alpha_s(\tan^{-1} \frac{1}{4} \sqrt{17}S) \]

\[ A_4 = 1 + \alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) - \alpha_s(0^\circ, 3S) - \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S) \]

\[ A_5 = \alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) - \alpha_s(0^\circ, 3S) - \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S) \]

\[ A_6 = \alpha_s(0^\circ, S) + \alpha_s(45^\circ, \sqrt{2}S) - \alpha_s(0^\circ, 3S) - \alpha_s(\tan^{-1} \frac{1}{3} \sqrt{10}S) \]
Again, for total load of 100%, the load distribution ratios \( F_1, F_2 \) and \( F_3 \) for 2 \( \Box \) piles can be calculated using Eqs. (18), (19) and (20).

\[
4 \cdot F_1 + 4 \cdot F_2 + 4 \cdot F_3 = 1
\]

For the normalized load distributions of the 3 \( \Box \), 3 \( \Box \) and 4 \( \Box \) pile groups, similar derivations can be found in Chang and Lin (2005).

**NUMERICAL OBSERVATIONS**

Based on numerical example of a 2 \( \Box \) pile foundation where \( D=0.9\) m and \( S/D=3 \), and the piles are locating in a soil layer where \( V_s=120 \) m/sec and \( \beta=5\% \), the load distribution ratio spectra and their time histories at the piles are obtained and plotted in Figure 4. It is found that the loads would vary from pile to pile when the operation frequency is less than 30 Hz. The static load ratios for the trailing, the middle, and the leading piles are approximately 25.6\%, 28.8\% and 45.6\%. Again, the leading piles may carry the largest static loads, and the trailing piles will have the smallest static loads. 3D variations of these load distribution ratios are plotted for the pile group in Figure 5 for operating frequencies of 0, 5 and 30 Hz. Time domain transformed values of these load ratios are about the same of all the piles. It implies that for arbitrary transient loads, the load would distribute evenly on the piles. This observation is found similar to those reported by Brown et al. (1987, 2001).

By defining \( K'/K_s \) as the ratio of the pile impedances calculated with and without the pile-to-pile interactions, one can conveniently examine the frequency-dependent pile-to-pile interaction effects on individual piles. Figure 6 depicts these ratio spectra for the above 2 \( \Box \)
piles. It is obvious to see that the trailing piles would be affected considerably by the pile-to-pile interaction under the static load, while the leading piles would only have least affection. However, at about 2.5 Hz, the pile foundation behaves oppositely. The impedance ratio of the trailing piles would be amplified to 4.95, and that of leading piles becomes 2.7. This figure shows clearly that the static load distributions of the pile foundation are essentially important in design practice. Table 2 summaries the static load distributions for 2 2, 2 3, 3 2, 3 3 and 4 3 pile foundations where the material properties are kept the same, and the ratio of the pile spacing to the pile diameter is varied at 3~6. It is plain to see that when the spacing ratio increases, the loads of the leading piles will slightly decrease and transfer to the trailing piles.

**COMPARISONS AND DISCUSSIONS**

The experimental results presented by Brown et al. (1988) on a full-scaled 3 3 pile foundation are taken to compare with the solutions of this study. As it is shown in Figure 7, the 12.8 m long steel pipe piles, having exterior diameter in 27.3 cm and thickness of the pipe wall in 0.927 cm, are free to translate at the pile head. The pile-to-pile spacing is kept three times as wide as the pile diameter. The ground surface to the depth of 2.8m consists of medium dense sand (D = 50%) where the firm clayey soil lies underneath it. The averaged unit weight of the soils is 1.57 T/m³. Young’s Modulus of the soils is calculated as 13.93 Mpa from the SPT-N values of the site (Table 3 shows the calculation basis). Assuming that the soil Poisson’s ratio is 0.3 and the material damping-ratio of soil is 5%, loading distribution ratios and their 3D variations for these pile groups from the proposed analysis are computed and plotted in Figures 8 and 9. Static load distributions on the leading piles are found to be 44%, and the ones on middle and trailing piles will be 35% and 21%, respectively. These results are again found comparative to those reported by Brown et al. (1988) and the recent
discussions made by Ilyas et al. (2004) and Rollins et al. (2005). The pile loads in the
transverse direction will increase from the center to the edge.

The numerical results obtained by Chae et al. (2004) using the 3D FEM analysis on a
scaled model of a 2 Ø pile foundation will be compared next. In their report, the 0.6 m long
steel pipe piles were installed in sands whereas the embedded pile length is 0.5 m. The
outside diameter of the pipe pile is 0.1 m, and the thickness of pipe wall is 3 mm. The
Young’s modulus of the soil is 25.8 Mpa, and the Poisson’s ratio is 0.3. The pile spacing was
kept 2 and 4 times as wide as the pile diameter, and the piles were thought to be free to
translate. Their results are shown in Figure 10, and it is crucial to notice that the dash lines
with open triangles are corresponding to the leading piles where the pile-to-pile spacing is
kept twice as the pile diameter. The ultimate load ratio is about 64%. The dash lines with
open squares are corresponding to the leading piles where the pile-to-pile spacing is four
times as wide as the pile diameter, in which the ultimate load ratio approximates 56%. The
solid lines are corresponding to the fixed pile head condition, in which higher resultant loads
were expected.

In premising that the material damping-ratio of the soil is 5%, the calculated load
distribution ratio spectra of these piles can thus be obtained and shown in Figures 11 and 12.
The results, without exception, are congruent with the prior. Large pile-to-pile spacing will
reduce the influences of the pile interactions. Variations of the loads are rare at much higher
frequencies (>100 Hz) compared to those shown in Figures 4 and 8. This is because that the
load rate dependence is changed with the soil stiffness and the pile orientation. Moreover,
one would imagine that the discrepancies between the above comparisons were getting much
more obvious as the soil became softer. In that case, the complexities of the soil compliance
along the piles will become a more important influence to the load distributions.
SUMMARY AND CONCLUSIONS

Despite that the soil compliance along the piles may affect the load distribution of a lateral loaded pile foundation, the dynamic pile-to-pile interaction factor that follows the elasticity theory for homogeneous medium is found capable to approximate the lateral load distributions of the grouped piles. A mirror pile group approximation method is used to model the load distributions for the grouped piles. Analytical equations to solve the load distributions were presented and discussed for $2 \Omega_2$ and $2 \Omega_3$ grouped piles. By comparing the results to the experimental and numerical observations from other researchers, this study suggests that its predictions for the cases of $2 \Omega_2$ and $3 \Omega_3$ pile foundations would be quite apt. However, ignoring the cap-pile-soil interactions would overestimate the load influences through the soils. Looking at the results from the modeling, the load distributions on the piles will depend on the orientation and spacing as well as diameter of the piles. In addition to that, the load frequency and the soil stiffness could affect the load distributions too. Variation of the load distribution is found apparent when the pile foundation is subject to steady state loads up to 30 Hz, which is typically higher than the frequencies of design interest. The load distribution ratios suggested in this paper should be applied with further caution because of theses simplicities. Nevertheless it can be seen from this modeling, the leading piles would preserve large loads whereas the trailing piles would sustain the least. In the direction perpendicular to the load, the center pile would carry the least load. Engineers should consider the load distributions in designing the pile foundations.

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### Table 1  Load Distribution Ratios of 2×2 and 3×3 piles with Static Horizontal Load Applied at Center of Foundation (S/D = 3)

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<th>3×3</th>
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<td>Arbirtrary pile</td>
<td>25%</td>
<td>13.2%</td>
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### Table 2  Lateral Load Distribution Ratios of Various Pile Groups (V_s = 120m/sec, D = 0.9m)

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<th>Orientation</th>
<th>3×3</th>
<th>4×4</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/D Location</td>
<td>trailing</td>
<td>leading</td>
</tr>
<tr>
<td>3</td>
<td>21%</td>
<td>34%</td>
</tr>
<tr>
<td>4</td>
<td>4%</td>
<td>17%</td>
</tr>
<tr>
<td>5</td>
<td>5%</td>
<td>18%</td>
</tr>
<tr>
<td>6</td>
<td>24.3%</td>
<td>34.8%</td>
</tr>
<tr>
<td>5.7%</td>
<td>18.6%</td>
<td>9.4%</td>
</tr>
<tr>
<td>6</td>
<td>25.2%</td>
<td>34.6%</td>
</tr>
</tbody>
</table>

| 6.2% | 19% | 9.6% | 25% | 11.6% | 28.6% | 6.8% | 12.4% | 7% | 13% | 9.6% | 14.4% | 16.2% | 20.6% |
Table 3  Calculations for Averaged Young’s Modulus of the Soils

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Thickness (m)</th>
<th>Soil Type</th>
<th>SPT-N</th>
<th>Corresponding Young’s modulus (kN/m²) (Bowles, 1982)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>Sand</td>
<td>5.0</td>
<td>10000.0</td>
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<tr>
<td>1.1</td>
<td>0.5</td>
<td>Sand</td>
<td>25.0</td>
<td>20000.0</td>
</tr>
<tr>
<td>2.0</td>
<td>0.9</td>
<td>Sand</td>
<td>55.0</td>
<td>35000.0</td>
</tr>
<tr>
<td>2.8</td>
<td>0.8</td>
<td>Sand</td>
<td>60.0</td>
<td>37500.0</td>
</tr>
<tr>
<td>3.0</td>
<td>0.2</td>
<td>Clay</td>
<td>10.0</td>
<td>8000.0</td>
</tr>
<tr>
<td>4.0</td>
<td>1.0</td>
<td>Clay</td>
<td>12.0</td>
<td>8640.0</td>
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<tr>
<td>5.4</td>
<td>1.4</td>
<td>Clay</td>
<td>15.0</td>
<td>9600.0</td>
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<tr>
<td>7.0</td>
<td>1.6</td>
<td>Clay</td>
<td>13.0</td>
<td>8960.0</td>
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<td>8.5</td>
<td>1.5</td>
<td>Clay</td>
<td>5.0</td>
<td>6400.0</td>
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<tr>
<td>10.5</td>
<td>2.0</td>
<td>Clay</td>
<td>18.0</td>
<td>10560.0</td>
</tr>
</tbody>
</table>

\[ E_{avg} = \frac{\sum E_i H_i}{\sum H_i} = 13927 \text{ kN/m}^2 \]

Figure 1  Shadowing Effects of Laterally Loaded Pile Foundation

(from Rollin et al., 1998)
Figure 2 Load Distribution Ratio Spectra and Corresponding Time Histories for Centrally loaded 3 \( \mathfrak{B} \) Piles \( (S/D = 3) \)
Figure 3 Actual Pile Group and Mirror Part in Solving Lateral Pile Load Distributions
(a) 2 piles (b) 3 piles
Figure 4  Load Distribution Ratio Spectra and Corresponding Time Histories for Laterally Loaded 2 3 Piles ($S/D = 3$)
Figure 5 Load Distribution Ratios for 2 pile group at Various Load Rates ($S/D = 3$)
Figure 6 Ratio Spectra of Pile Impedances Calculated w/ and w/o Pile-to-Pile Interactions on Laterally Loaded 2 1B Grouped Piles
Figure 7 Testing and Site Conditions and Load Displacement Results (from Brown et al., 1988)
Figure 8 Load Distribution Ratio Spectra and Time Histories for Lateral Loaded 3 \[ \frac{D}{H} \] Piles \((S/D = 3)\)
Figure 9 Load Distribution Ratios for 3 Ø pile group at Various Load Rates ($S/D = 3$)
Figure 10 Load Distribution Ratios versus Pile Displacements (after Chae et al., 2004)
Figure 11  Load Distribution Ratio Spectra and Corresponding Time Histories for Laterally Loaded 2 \( \Omega \) Piles \((S/D = 2)\)
Figure 12  Load Distribution Ratio Spectra and Corresponding Time Histories for Laterally Loaded 2 \( \mathcal{P} \) Piles \( S/D = 4 \)